Assumption of nonvanishingness of vacuum expectation of the scalar field for spontaneous symmetry breaking is superfluous

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Abstract

For spontaneous breaking of global or gauge symmetry, it is superfluous to assume that the vacuum expectation value of the scalar field manifesting the symmetry is nonvanishing. The vacuum with spontaneous symmetry breaking simply corresponds to the nonzero number of particles of one or more components of the real scalar field.

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A crucial ingredient of the theories of spontaneous symmetry breaking is the assumption that the vaccum expectation value of the scalar field manifesting the the symmetry, $\overline{\phi}$, is nonzero [1–6]. This conflicts the basic fact that the Hilbert space is spanned by states with definite number of particles and/or antiparticles in each mode. In fact, $\overline{\phi} \neq 0$ is only a sufficient but not a necessary condition for the required results. Though it is widely acknowledged as a property of vacuum, here we point out that this assumption is superfluous. For spontaneous gauge symmetry breaking in superfluidity or superconductivity, it has been pointed out that the nonvanishingness of the expectation value of the field operator is by no means essential but an approximate though convenient approach [7].

We follow the notations and line of development in [1], and start with Goldstone theorem. Under a continuous symmetry which transforms a set of Hermitian scalar fields $\phi_n(x)$ as

$$\phi_n(x) \rightarrow \phi_n(x) + i\epsilon \sum_m t_{nm}\phi_m(x), \tag{1}$$

where it_{nm} is the finite and real matrix corresponding to the symmetry transformation. Consequently, the invariance of the action and measure, and thus the effective potential leads to

$$\sum_{nm} \frac{\partial V(\phi)}{\partial \phi_n} t_{nm}\phi_m = 0, \tag{2}$$

therefore at the minimum the $V(\phi)$,

$$\sum_{nm} \Delta^{-1}_{nl}(0) t_{nm}\phi_m = 0, \tag{3}$$

where $\Delta^{-1}_{nl}(0) = \partial^2 V(\phi)/\partial\phi_n \partial\phi_l$ is the reciprocal of the momentum space propagator. In the conventional approach, the indication of symmetry breaking is $\sum_m t_{nm}\overline{\phi_m} \neq 0$ obtained from (1). By calcualting the vacuum expectation of (3), one obtains Goldstone theorem, i.e. $\sum_m t_{nm}\overline{\phi_m} \neq 0$ implies the massless boson. There is one massless boson for every independent broken symmetry.

Now we insist that $\overline{\phi_m}$, and thus the expectation of each term in (1) vanishes, therefore the expectation of (3) is trivially satisfied. To examin the consequence of symmetry breaking,

one should study another operator which is a functional of ϕ_n and can also reflect the symmetry of the Hamiltonian, or say the effective potential. This is just $\phi_n\phi_n$. For instance, in the classical example

$$\mathcal{L} = -\frac{1}{2}\sum_n \partial_\mu\phi_n\partial^\mu\phi_n - \frac{\mathcal{M}^2}{2}\sum_n \phi_n\phi_n - \frac{g}{4}(\sum_n \phi_n\phi_n)^2, \tag{4}$$

which is invariant under the group $O(N)$, the effective potential is a functional of $\phi_n\phi_n$.

Under the transformation (1), $\phi_n\phi_n$ transforms as

$$\phi_n(x)\phi_n(x) \rightarrow \phi_n\phi_n(x) + 2i\epsilon\sum_m t_{nm}\phi_n(x)\phi_m(x). \tag{5}$$

If the symmetry is not broken, $\overline{\phi_n\phi_n}$ remains unchanged. Since $\overline{\phi_n\phi_m} \equiv 0$ for $n \neq m$, (5) implies $\overline{\phi_n\phi_n} \rightarrow (1+2\epsilon i)\overline{\phi_n\phi_n}$, therefore $\overline{\phi_n\phi_n}$ and thus the effective potential is invariant if and only if $\overline{\phi_n\phi_n} = 0$.

On the other hand, the mode expansion

$$\phi_n = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}(a_k^{(n)}e^{ikx} + a_k^{(n)\dagger}e^{-ikx}) \tag{6}$$

with $[a_k^{(n)}, a_{k'}^{(n)\dagger}] = (2\pi)^3\delta^{(3)}(k-k')$ yields

$$\overline{\phi_n\phi_n} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\overline{N_k^{(n)}} + C, \tag{7}$$

where $N_k^{(n)} = a_k^{(n)\dagger}a_k^{(n)}$, $C = \int d^3k/2\omega_k$ is an infinite c-number. In the derivation, we exploited the property that $\overline{a_k^{(n)}a_k^{(n)}} = 0$ and $\overline{a_k^{(n)}a_{k'}^{(n)\dagger}} = 0$ for $k \neq k'$. Note that the value of $\overline{\phi_n\phi_n}$ is independent of x.

The least value of (7) correponds to $N_k^{(n)} = 0$. To reconcile with $\overline{\phi_n\phi_n} = 0$ in the absence of symmetry breaking, the term of infinite c-number should be ignored. As well known, the same problem and strategy appear in the energy calculated from (6). In conclusion, the vacuum without symmetry breaking is simply the state in which there is no particle and thus $\overline{\phi_n\phi_n} = 0$.

Eq. (5) implies that spontaneous symmetry breaking corresponds to $\sum_m t_{nm}\overline{\phi_n\phi_m} \neq 0$, therefore $\overline{\phi_n\phi_n} \neq 0$, it is random for which n this enequality holds, as implied by symmetry.

Multiplying (3) by $\sum_n \phi_n$ and considering $\overline{\phi_n\phi_m} \equiv 0$ for $n \neq m$, we obtain

$$\sum_{nm} \Delta_{nl}^{-1}(0) t_{nm} \overline{\phi_m\phi_m} = 0. \tag{8}$$

Therefore whenever there is a component k for which $\overline{\phi_k\phi_k}$ is nonzero, the summation $\sum_m t_{nm}\overline{\phi_m\phi_m}$ which includes $\overline{\phi_k\phi_k}$ must be nonzero sice each $\overline{\phi_m\phi_m} \leq 0$. The nonvanishing $\sum_m t_{nm}\overline{\phi_m\phi_m}$ should be an eigenvector of $\Delta_{nl}^{-1}(0)$ with eigenvalue zero, thus $\Delta_{nl}(q)$ has a pole at $q^2 = 0$, i.e. there is a massless boson. There is one massless boson for every independent broken symmetry. All essential results of Goldstone theorem can thus obtained.

For the example (4), the minimum of the effective potential is at

$$\sum_n \overline{\phi_n\phi_n} = -\frac{\mathcal{M}^2}{g}, \tag{9}$$

the mass matrix is

$$\begin{aligned} M_{nm}^2 &= \overline{\frac{\partial^2 V(\phi)}{\partial\phi_n\partial\phi_m}} \\ &= 2g\overline{\phi_n\phi_n}\delta_{nm}. \end{aligned} \tag{10}$$

$O(N)$ symmetry is broken down to $O(N-1)$ when there is one component ϕ_1 with $\overline{\phi_1\phi_1} \neq 0$ while $\overline{\phi_i\phi_i} = 0$ for $i \neq 1$, therefore $\overline{\phi_1\phi_1} = -\mathcal{M}^2/g$. Consequently there is one massive boson with mass $2|\mathcal{M}^2|$ and $N-1$ massless bosons. This is consistent with the general argument above. The massless bosons correpond to the $O(N-1)$ symmetry relating different possible selection of ϕ_1.

From the general viewpoint of quantum mechanics, if initially the state of a system is in an eigenstate of a relevant operator or a set of operators commuting each other, it will always be in this stationary state if these operators commute the Hamiltonian, or in a nearly-stationary state if there is near-degeneracy in case the relevant operator does not commute the Hamiltonian. The initial condition is determined by a basic postulate that the measurement projects the state to an eigenstate of the relevant operator *defining* the physical situation. We think this is the essence of various spontaneous symmetry breaking [7]. In the present case, the relevant operators are $\phi_n\phi_n$, which commute the effective

potential. Therefore spontaneous summetry breaking can occur. Indeed, $\phi_n\phi_n$ represents the particle numbers in all modes, and the vacuum or excited state is just defined through the particle number. Of course, as well known, the results also applies if the symmetry is broken non-spontaneously. The vacuum is nothing but the ground state, with least total number of particles, spontaneous symmetry breaking makes the state of the system in one of the degenerate ground states instead of the combination, therefore the number of particles of one or more components of the scalar field are nonzero, In the meantime, massless bosons are yielded with number determined by the broken symmetry.

In the gauge symmetry breaking, the gauge field acquires a mass, this can also be obtained from the present argument. Defining $v_n = \sqrt{\overline{\phi_n(0)\phi_n(0)}}$, the new fields $\tilde{\phi}(x)$ in the unitary gauge can be obtained by $\sum_{nm}\tilde{\phi}(x)t_{nm}v_m = 0$. Substituting the shift field $\phi'_n = \phi_n - v_n$ to the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sum_n(\partial_\mu\tilde{\phi}_n - i\sum_{m\alpha}t^\alpha_{nm}A_{\alpha\mu}\tilde{\phi}_m)^2, \tag{11}$$

the gauge boson masses $\mu^2_{\alpha\beta} = -\sum_{nml}t^\alpha_{nm}t^\beta_{nl}v_m v_l$ is yielded in the same way as the previous approach. The proof of renormalizability is also valid with the new definition of v_n here. In the $SU(2)\times U(1)$ electroweak theory, the doublet scalar field can be in the form of

$$\phi = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \tag{12}$$

with ϕ_0 hermitian. Defining the shift fields as

$$\phi = \begin{pmatrix} 0 \\ \sqrt{\overline{\phi_0\phi_0}} \end{pmatrix} + \begin{pmatrix} \phi'_+ \\ \phi'_0 \end{pmatrix}, \tag{13}$$

all the previous results are yielded in the similar way.

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REFERENCES

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[1] S. Weinberg, *The Quantum Theory of Fields, Vol. II* (Cambridge University, Cambridge, 1996).

[2] J. Goldstone, Nuovo Cimento **19**, 154 (1961); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).

[3] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[4] P.W. Higgs, Phys. Lett. **12**, 132 (1964); Phys. Lett. **13**, 508 (1964); Phys. Rev. **145**, 1156 (1966); F. Englert & R. Brout, Phys. Lett. **13**, 321 (1964); G.S. Guralnilk, C.R. Hagen, and T.W.B. Kibble, Phys. Lett. **13**, 585 (1964).

[5] G. t'Hooft, Nucl. Phys. B **35**, 167 (1971); B.W. Lee, Phys. Rev. D **5**, 823 (1972); K. Fujikawa, B. W. Lee, and A. Sanda, Phys. Rev. D **6**, 2923 (1972).

[6] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Physics*, edited by N. Svartholm, (Almqvist & Wiksells, Stockholm, 1968), P. 9367.

[7] Y. Shi, LANL Report No. quant-ph/9702026.